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Hoop Conjecture and the Horizon Formation Cross Section in Kaluza-Klein Spacetimes

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We analyze momentarily static initial data sets of the gravitational field produced by two-point sources in five-dimensional Kaluza-Klein spacetimes. These initial data sets are characterized by the mass, the separation of sources and the size of a extra dimension. Using these initial data sets, we discuss the condition for black hole formation, and propose a new conjecture which is a hybrid of the four-dimensional hoop conjecture and the five-dimensional hyperhoop conjecture. By using the new conjecture, we estimate the cross section of black hole formation due to collisions of particles in Kaluza-Klein spacetimes. We show that the mass dependence of the cross section gives us information about the size and the number of the compactified extra dimensions.

I. INTRODUCTION

Classical theory of gravity in higher dimensions has gathered much attention since the brane world, which suggests the possibility of large extra dimensions, has been proposed [1, 2]. Black holes in this framework would be believed as key objects for verification of extra dimensions. It has been clarified that higher-dimensional black holes in asymptotically flat spacetimes have richer structure than four-dimensional black holes [3–6] (see also [7]). It was also suggested that higher-dimensional mini-black holes might be produced in accelerators [8–14] and in cosmic ray events [15–17]. Such black holes, which would evaporate by the Hawking radiation, are expected to play crucial roles in the development of the quantum theory of gravity.

In this paper, we focus on the black hole formation rate in higher-dimensional spacetimes. The hoop conjecture, proposed by Thorne[18], gives a criterion for the black hole formation in four-dimensional spacetimes. It is thought that the criterion by conjecture can be applied to a variety of the black hole formation processes. However, as will be mentioned in Sec.III A, the existence of black string solutions means that Thorne’s hoop conjecture, where the length of one-dimensional loops are used to measure the compactness of a system, is not true in higher dimensions. In higher-dimensional spacetimes, the hyperhoop conjecture has been proposed as the condition for black hole formation [19–21]. In the hyperhoop conjecture, the area of codimension three closed surfaces is used instead of the length of one-dimensional loops.

Apparent horizon formation is analyzed in the collision of two-point particles, and then higher-dimensional black hole formation rates in accelerators has been predicted [22–24]. These works are concentrated on the cases in which spacetimes have asymptotically Euclidean spatial sections. The assumption of the asymptotically Euclidean spatial sections is likely to be relevant if the black hole size is much smaller than the size of extra dimensions. On the other hand, if the size of the extra dimensions are comparable to the size of black hole, the formation rate would be changed. Does the black hole formation rate give us any

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information about the size of the extra dimensions?

In an asymptotically flat D -dimensional spacetime with Euclidean spatial sections, a typical black hole has the horizon radius $\sim (G_D M)^{1/(D-3)}$, where G_D and M are the gravitational constant and the mass of the black hole, respectively. In this case, the mass dependence of the cross section for black hole formation σ_p in the collision of particles is¹

$$\sigma_p \propto M^{2/(D-3)}, \quad (1)$$

(see Ref. [25] as a recent review, and references therein). If the space has compactified directions, this dependence should be modified. Suppose that n directions in the D -dimensional spacetime are compactified into a length scale l . If $l \gg (G_D M)^{1/(D-3)}$, the mass dependence of σ_p is the same as the case of asymptotically flat spacetimes with Euclidean spatial sections. On the other hand, if $l \ll (G_D M)^{1/(D-3)}$, we expect that the compactified dimensions can be neglected, and σ_p is given by the horizon radius of a typical $(D-n)$ -dimensional black hole $(G_{D-n} M)^{1/(D-n-3)}$. Namely, we expect that the mass dependence of σ_p behaves as

$$\sigma_p \propto M^{2/(D-n-3)}. \quad (2)$$

This transition of the mass dependence of σ_p might give us the information about the compactification scale.

In this paper, we consider systems of two-point particles in a five-dimensional Kaluza-Klein spacetime. We use, concretely, the four-dimensional Euclidean Taub-NUT space [26], which has a twisted S^1 as the extra dimension. We construct initial data sets of the gravitational field around two-point particles including the parameter which describes the separation of the particles. As varying the separation parameter, we inspect the existence of a cover-all apparent horizon using the same technique as is used in Ref. [27]. We will show that there is the maximum separation parameter for the existence of a cover-all apparent horizon, and we consider this apparent horizon indicates the necessary and sufficient compactness of the system for the black hole formation. From the shape of this apparent horizon, we will obtain the condition for the black hole formation in the higher-dimensional spacetime with the compactified extra dimension.

¹ In brane world scenarios, since particles of matter are confined on a three-brane, it would be useful to consider the cross section which has the dimension of (length)².

For the following reason, we consider the four-dimensional Taub-NUT space, which is a twisted S^1 fiber bundle over the flat three-dimensional base space, as a time slice, not a direct product of S^1 and the base space. Let us consider a spherically symmetric black hole in a five-dimensional asymptotically flat spacetime with Euclidean spatial sections. The geometry admits $SO(4)$ spatial isometry. If we impose a periodic identification in a spatial direction which causes S^1 compactification of a direct product, the isometry reduces to $SO(3)$. In contrast, a black hole can have $SO(3) \times U(1)$ symmetry if it is in a five-dimensional spacetime where the extra dimension is the twisted S^1 fiber over the four-dimensional spacetime [28–30]. Similarly, in the systems of two-point sources, the symmetry is $U(1)$ in the direct product spaces while it can be $U(1) \times U(1)$ in the twisted S^1 bundle cases. The spaces with twisted S^1 bundle structure can have more symmetry than the simple direct product spaces. Using this advantage, recently, black hole solutions with nontrivial asymptotic structure are studied in the five-dimensional Einstein-Maxwell theory [31–40]. This advantage makes it possible to search for apparent horizons by solving ordinary differential equations in the space with twisted S^1 bundle structure[27].

In a five-dimensional Kaluza-Klein spacetime, we propose the new condition of horizon formation which is a hybrid of the four-dimensional hoop conjecture and the five-dimensional hyperhoop conjecture. Extrapolating the new proposal to general situations, we estimate the cross section of the black hole formation in collision of particles as a function of the mass scale in any dimension. We show that the mass dependence of the cross section changes when the mass scale becomes comparable to the scale of the extra dimension.

The organization of the paper is as follows. The method for constructing the initial data sets is shown in Sec.II. In Sec.III, the hyperhoop conjecture in the spacetime with a compactified extra dimension is examined, and a new conjecture is proposed. Effects of the compactification size of the extra dimensions on the black hole production cross section are discussed in Sec.IV, and summary and discussions are given in Sec.V.

II. MOMENTARILY STATIC INITIAL DATA IN KALUZA-KLEIN SPACES

In this section, as a preparation to discuss the hyperhoop conjecture in Kaluza-Klein spaces, we construct initial data sets for two-point sources with a compactified extra dimension, and discuss geometrical properties.

A. Construction of initial data

Let us consider an initial data set of the induced metric and the extrinsic curvature (h_{ij}, K_{ij}) on a four-dimensional spacelike hypersurface Σ , which satisfies the Hamiltonian and momentum constraints,

$$\mathcal{R} - K_{ij}K^{ij} + K^2 = 16\pi G_5 \rho_m, \quad (3)$$

$$D_j (K^{ij} - h^{ij}K) = 8\pi G_5 J_m^i, \quad (4)$$

where ρ_m and J_m^i are the energy density and the energy flux of matter, and D_i and \mathcal{R} are the covariant derivative within Σ and the scalar curvature with respect to h_{ij} .

We restrict ourselves to momentarily static cases, i.e.,

$$K_{ij} = 0, \quad (5)$$

and assume the induced metric has the form of

$$h_{ij}dx^i dx^j = F^2 ds_{\text{RF}}^2, \quad (6)$$

where ds_{RF}^2 is a Ricci flat metric. In this case, the vacuum momentum constraint is trivially satisfied and the vacuum Hamiltonian constraint reduces to

$$\Delta_{\text{RF}} F = 0, \quad (7)$$

where Δ_{RF} is the Laplace operator of the Ricci flat metric.

For the purpose of considering two-point sources in a Kaluza-Klein space, we take the two-center Taub-NUT metric, which is Ricci flat. The metric in the Gibbons-Hawking(GH) form[26] is given by

$$ds_{\text{GH}}^2 = V^{-1} ds_{\text{3dE}}^2 + \frac{V}{4} l^2 (d\psi + \omega_\phi d\phi)^2, \quad (8)$$

$$ds_{\text{3dE}}^2 = dr^2 + r^2 d\theta^2 + r^2 \sin^2 \theta d\phi^2, \quad (9)$$

$$V^{-1} = 1 + \frac{l}{2} \left(\frac{1}{\sqrt{r^2 + a^2 - 2ar \cos \theta}} + \frac{1}{\sqrt{r^2 + a^2 + 2ar \cos \theta}} \right), \quad (10)$$

$$\omega_\phi = \frac{r \cos \theta - a}{\sqrt{r^2 + a^2 - 2ar \cos \theta}} + \frac{r \cos \theta + a}{\sqrt{r^2 + a^2 + 2ar \cos \theta}}. \quad (11)$$

The range of angular coordinates are $0 \leq \theta \leq \pi$, $0 \leq \phi \leq 2\pi$ and $0 \leq \psi \leq 4\pi$. We consider two-point sources locate at the two centers of (8) which are fixed points of the action of

isometry generated by the Killing vector ∂_ψ . Then, we can assume that the function F has the same symmetry, i.e., F does not depend on ψ . In this case, the Eq.(7) reduces to

$$\Delta_{3\text{dE}} F = 0, \quad (12)$$

where $\Delta_{3\text{dE}}$ is the Laplace operator on the three-dimensional Euclidean metric of (9). A solution of (12) for two-point sources is

$$F = 1 + \frac{m_1/l}{\sqrt{r^2 + a^2 - 2ar \cos \theta}} + \frac{m_2/l}{\sqrt{r^2 + a^2 + 2ar \cos \theta}}. \quad (13)$$

In the limit $r \rightarrow \infty$, we have $F \rightarrow 1$, $V^{-1} \rightarrow 1$, $\omega_\phi \rightarrow 2 \cos \theta$, then we can see the asymptotic form of h_{ij} as

$$h_{ij} dx^i dx^j \rightarrow dr^2 + r^2(d\theta^2 + \sin^2 \theta d\phi^2) + l^2 \left(\frac{d\psi}{2} + \cos \theta d\phi \right)^2. \quad (14)$$

Thus, we can regard the extra dimension, twisted S^1 spanned by ψ , is compactified in the size l at the asymptotic region. An $r = \text{constant}$ surface of the space with the metric (14) is homeomorphic to the lens space $L(2; 1) = S^3/\mathbb{Z}_2$ [27, 36, 38].

The Abbott-Deser mass[41] of the initial metric (6) with (8)-(11) and (13) can be calculated as

$$G_5 M = 3\pi(m_1 + m_2), \quad (15)$$

where we have used the metric

$$ds^2 = \left(1 + \frac{l}{r}\right) (dr^2 + r^2 d\theta^2 + r^2 \sin^2 \theta d\phi^2) + l^2 \left(1 + \frac{l}{r}\right)^{-1} \left(\frac{d\psi}{2} + \cos \theta d\phi\right)^2, \quad (16)$$

which is the $a = 0$ case of (8), as the reference metric². For simplicity, hereafter, we set

$$m_1 = m_2 = m. \quad (17)$$

The mass parameter m has the dimension of length square. Hereafter, a nondimensional parameter m/l^2 is the key parameter.

² The reference metric is different from the one used in Refs.[42–44] because the topology of $r = \text{const.}$ surface at infinity is not S^3 but S^3/\mathbb{Z}_2 in our case. The reference metric is the same as the induced metric of a static slice in the Gross-Perry-Sorkin(GPS) monopole solution [45, 46] except for the factor $1/2$ in front of $d\psi$.

B. Apparent horizon

If a spacetime is an asymptotically predictable spacetime from a Cauchy surface, and the null energy condition is satisfied, then the existence of an apparent horizon guarantees the existence of an event horizon [47]. Then, the existence of an apparent horizon is a relevant indicator for the formation of a black hole.

Before considering the $a \neq 0$ cases, it is useful to see the $a = 0$ case, where we can calculate the horizon radius analytically. Putting $a = 0$ in (6) with (8)-(11) and (13), we have the induced metric in the form

$$h_{ij}dx^i dx^j = \left(1 + \frac{2m}{lr}\right)^2 \left[\left(1 + \frac{l}{r}\right) (dr^2 + r^2 d\theta^2 + r^2 \sin^2 \theta d\phi^2) + \left(1 + \frac{l}{r}\right)^{-1} l^2 \left(\frac{d\psi}{2} + \cos \theta d\phi\right)^2 \right]. \quad (18)$$

Then, due to the symmetry, the horizon is given as an $r = \text{const.}$ surface. The area of an $r = \text{const.}$ three-dimensional surface $A(r)$ is given by

$$A(r) = 8\pi^2 F^3 V^{-1/2} r^2 l = \frac{8\pi^2 \sqrt{l+r} (2m+lr)^3}{l^2 r^{3/2}}. \quad (19)$$

Since the initial hypersurface is momentarily static, i.e., $K_{ij} = 0$, the apparent horizon is a minimal surface. Then, we can obtain the horizon radius r_h as a solution of the equation

$$\left. \frac{dA(r)}{dr} \right|_{r=r_h} = 0. \quad (20)$$

The horizon radius r_h is given by

$$r_h = \frac{1}{8l} \left[-3l^2 + 4m + \sqrt{9l^4 + 72ml^2 + 16m^2} \right] \quad (21)$$

and evaluated as

$$r_h \simeq \begin{cases} \frac{m}{l} & \text{for } \frac{m}{l^2} \gg 1, \\ \frac{2m}{l} & \text{for } \frac{m}{l^2} \ll 1. \end{cases} \quad (22)$$

In the $a \neq 0$ cases, the geometry given by (6) with (8)-(11) and (13) becomes less symmetric compared to the $a = 0$ case. But, it still has $U(1) \times U(1)$ isometry generated by the commuting Killing vectors ∂_ϕ and ∂_ψ . Then an apparent horizon, if there exists, should be given by the surface in the form

$$r = r_h(\theta). \quad (23)$$

This simplicity comes from the fact that we use the two-center Taub-NUT space as the base space. So, we have to solve the ordinary differential equation for minimal surfaces in the form

$$r_h'' - \frac{3r_h'^2}{r_h} - 2r_h + (r_h^2 + r_h'^2) \left\{ \frac{r_h'}{r_h^2} \cot \theta - \left(G_r(r_h, \theta) - \frac{r_h'}{r_h^2} G_\theta(r_h, \theta) \right) \right\} = 0, \quad (24)$$

where a prime means the derivative with respect to θ and functions $G_r(r, \theta)$ and $G_\theta(r, \theta)$ are

$$G_r(r, \theta) := \partial_r (3 \ln F + \frac{1}{2} \ln V^{-1}), \quad (25)$$

$$G_\theta(r, \theta) := \partial_\theta (3 \ln F + \frac{1}{2} \ln V^{-1}). \quad (26)$$

The apparent horizons, which are solutions of (24), are described by smooth closed curves in two-dimensional plane (r, θ) . Typical graphs of the apparent horizons are plotted in Fig.1. As shown in Fig.1, the separation of the two-point sources must be smaller than a certain value for the existence of a cover-all apparent horizon.

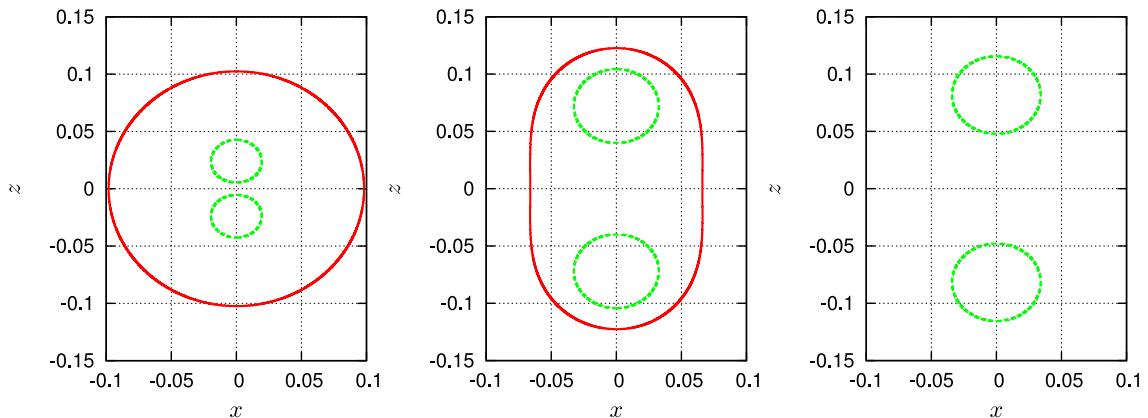


FIG. 1: Apparent horizons for $m^2/l = 0.0025$ and $a = 0.02m^{1/2}$, $0.07m^{1/2}$ and $0.08m^{1/2}$ from left to right. Solid lines represent cover-all horizons, and dashed lines represent each independent horizons of two-point sources. In the right panel, where $a = 0.08m^{1/2}$, no cover-all horizon exists. The horizontal and vertical axes represent $x = r \sin \theta$ and $z = r \cos \theta$, respectively.

III. TEST OF HYPERHOOP CONJECTURE IN KALUZA-KLEIN SPACES

A. Hyperhoop conjecture

The black hole production rate due to collisions of particles can be evaluated by using the notion of the hyperhoop conjecture for asymptotically Minkowski spacetimes in higher

dimensions[19–21]. Here we check whether this conjecture is true in the case of higher dimensions with compactified directions.

The hyperhoop conjecture is as follows: *Black holes with horizons form when and only when a mass M gets compacted into a region whose $(D - 3)$ -dimensional volume in every direction is*

$$V_{D-3} \lesssim \alpha_D G_D M, \quad (27)$$

where α_D is a numerical factor and G_D is the gravitational constant in D -dimensional theory of gravity, and the $(D - 3)$ -dimensional volume V_{D-3} means the volume of a $(D - 3)$ -dimensional closed submanifold(hyperhoop) of a spacelike hypersurface.

It should be noted that this conjecture has some ambiguities. The definitions of the mass and the hyperhoop are not explicitly given. In this paper, we interpret M as the total mass of a system, and V_{D-3} as a typical $(D - 3)$ -dimensional volume of a closed submanifold which represents the compactness of a system.

In Thorne’s original hoop conjecture, the one-dimensional circumference is used as an indicator of the compactness of a system. However, in the five-dimensional Einstein gravity, we know that the black string solutions have hoops with infinite length. In addition, D. Ida and K. Nakao showed that the one-dimensional circumference of the apparent horizon which is produced by a uniform line source can be infinitely long. Then, they proposed the hyperhoop which measures the compactness of the system [19].

If the extra dimensions have finite sizes, it is nontrivial whether the volume of the hyperhoop can give us the appropriate criterion for black hole formation or not. When the size of a black hole is much smaller than the size of extra dimensions, the hyperhoop conjecture would be true. However, when the black hole is as large as the extra dimensions, validity of the hyperhoop conjecture is not clear. Then, we check whether the hyperhoop works in Kaluza-Klein spaces in the next subsection.

B. Test of hyperhoop conjecture in Kaluza-Klein spaces

The authors in Ref.[20, 21] studied the criterion of black hole formation in relation to the compactness of explicit matter source distributions. In contrast, we consider systems consist of only two-point masses, in which geometrical information is only the distance between them. The (hyper) hoop conjecture claims that if a black hole exists, any length

scale characterizing the black hole should be less than a critical scale determined by the mass of the black hole. To check this statement, we use the size of a cover-all apparent horizon, if it exists, to measure the typical length scale of a black hole, and use the Abbott-Deser mass as the total mass M .

We have constructed the initial data of the gravitational field of two-point sources with the separation parameter a in the previous section. Now, we discuss the criterion of black hole formation by introducing a geometrical quantity $V(a)$, which measures the compactness of a cover-all apparent horizon. At present, we do not restrict the dimension of $V(a)$.

We require the inequality

$$V(a) \lesssim (\text{critical size}) \quad (28)$$

for cover-all apparent horizons if they exist, where the “critical size” is a quantity related to the Abbott-Deser mass.

According to the numerical calculations, there exists a critical value of separation parameter a_{cr} such that cover-all horizon exists if $a < a_{\text{cr}}$. Then, we can expect that $V(a)$ and the “critical size” satisfy following two properties:

- (i) $V(a)$ is a monotonic increasing function of a at least in the vicinity of a_{cr} .
- (ii) $V(a_{\text{cr}}) \sim (\text{critical size})$.

First, we check the hyperhoop conjecture in the form (27) for $D = 5$. The quantity $V(a)$ in the left-hand side of (28) is the area of a two-dimensional closed surface $V_2(a)$. We consider closed geodesic 2-surfaces \mathcal{A} on a cover-all horizon which characterize the shape of horizon. We take the surface which has maximum area among \mathcal{A} as $V_2(a)$.

We fix the “critical size” in (28), using the horizon radius of five-dimensional Schwarzschild black holes

$$r_{\text{Sch}} = \sqrt{\frac{8G_5 M}{3\pi}}. \quad (29)$$

Setting (28) holds equality in the case of Schwarzschild black holes, i.e.,

$$\text{critical size} = 4\pi r_{\text{Sch}}^2 = \frac{32}{3}G_5 M, \quad (30)$$

we have

$$V_2(a) \lesssim \frac{32}{3}G_5 M. \quad (31)$$

Then the numerical value of α in (27) is also fixed as $32/3$.

Because the geometries have the isometry group generated by ∂_ϕ and ∂_ψ , and the discrete isometry $\theta \rightarrow \pi - \theta$, we consider the following typical closed geodetic 2-surfaces \mathcal{A} on a horizon:

$$\mathcal{A}_{\theta=\pi/2} : \text{area of } \theta = \frac{\pi}{2} \text{ surface,} \quad (32)$$

$$\mathcal{A}_{\phi=0} : \text{area of } \phi = 0 \text{ surface,} \quad (33)$$

$$\mathcal{A}_{\psi=0} : \text{area of } \psi = 0 \text{ surface.} \quad (34)$$

As is noted before, we define

$$V_2(a) = \max \{ \mathcal{A}_{\theta=\pi/2}, \mathcal{A}_{\phi=0}, \mathcal{A}_{\psi=0} \}. \quad (35)$$

The values of \mathcal{A} 's are depicted as functions of a in some cases of m/l^2 in Fig.2. As can be seen in the Fig.2, $V_2 = \mathcal{A}_{\phi=0}$ in the $m/l^2 = 0.0025$ case and $V_2 = \mathcal{A}_{\psi=0}$ in the other cases. In the cases of $m/l^2 \gtrsim 1$, V_2 is not monotonic increasing function of a . We can see also $V_2(a_{\text{cr}})$ becomes much larger than the critical size, $32G_5M/3$. Namely, both of conditions (i) and (ii) are not satisfied. In contrast, we show that these two conditions are satisfied in the asymptotically Euclidean case in AppendixA.

This failure of the hyperhoop conjecture may be clear if we consider a direct product spacetime $S^1 \times M_{\text{Sch}}$, where M_{Sch} is a four-dimensional Schwarzschild spacetime. In this spacetime, when $G_5M/l^2 > 1$, V_2 is given by

$$V_2 \sim 16\pi(G_4M)^2 \sim 4G_5^2M^2/(\pi l^2), \quad (36)$$

where we have used the relation between G_4 and G_5 given by

$$G_4 \sim \frac{G_5}{2\pi l}. \quad (37)$$

Though $V_2/(G_5M)$ can be infinitely large for $G_5M/l^2 \gg 1$, a horizon exists for any G_5M/l^2 . This fact means the failure of the condition (ii).

It should be noted that we cannot give the complete counterexample for the hyperhoop conjecture in a nontrivial asymptotic structure in this paper. Because there are ambiguities in the definition of V_{D-3} and the mass which should be used in the Eq.(27), and also in the interpretation of the “ \lesssim ”. Nevertheless, we found the completely different feature from the asymptotically Euclidean case which suggests the hyperhoop conjecture cannot be extended straightforwardly to cases with finite sizes of extra dimensions.

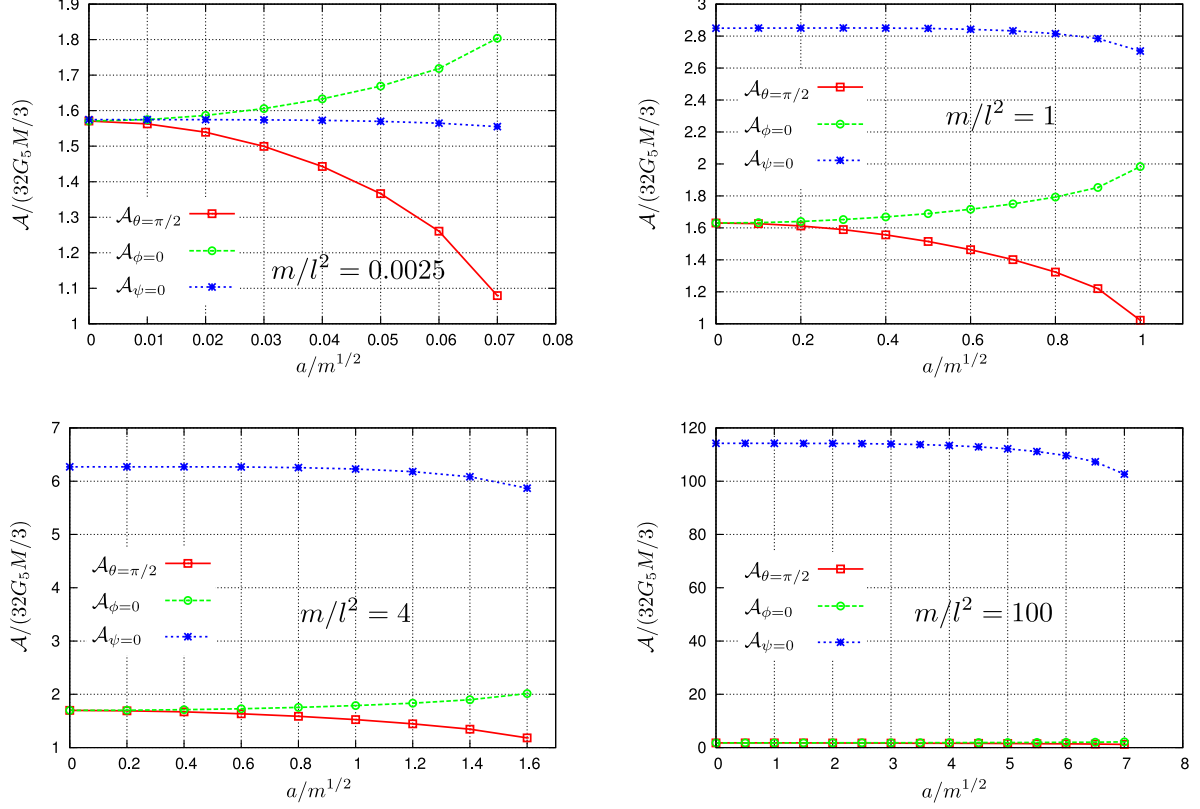


FIG. 2: Areas of the two-dimensional geodesic surfaces \mathcal{A} on apparent horizons are depicted as functions of $a/m^{1/2}$ in the cases of $m/l^2 = 0.0025, 1, 4$, and 100 . $V_2 = \mathcal{A}_{\psi=0}$, the maximum value of \mathcal{A} is not a monotonic increasing function of $a/m^{1/2}$ in the cases $m/l^2 = 1, 4, 100$. The critical values of $\mathcal{A}_{\psi=0}$ become much larger than the critical size $32G_5M/3$ in these cases.

C. Criterion for large black hole formation

Next, let us focus on the $m/l^2 \gg 1$ case, where V_2 is not appropriate for the left-hand side of the condition (27). In this case, the size of the black hole is much larger than that of the extra dimension, and the gravitational field outside the horizon is effectively four-dimensional. Then, we can expect the ordinary four-dimensional hoop conjecture

$$V_1 \lesssim 4\pi G_4 M \quad (38)$$

is true, where the constant α has been determined by using four-dimensional Schwarzschild black holes. Then we take one-dimensional hoop $V_1(a)$ as $V(a)$ in (28). Using Eq.(37), we

rewrite (28) as

$$V_1(a) \lesssim \frac{2G_5 M}{l}. \quad (39)$$

In order to estimate $V_1(a)$, we consider the following typical closed geodesic curves \mathcal{C} on a horizon:

$$\mathcal{C}_{\psi=0}^{\phi=0} : \text{length of } \phi = 0 \text{ and } \psi = 0 \text{ curve}, \quad (40)$$

$$\mathcal{C}_{\psi=0}^{\theta=\pi/2} : \text{length of } \theta = \frac{\pi}{2} \text{ and } \psi = 0 \text{ curve}, \quad (41)$$

$$\mathcal{C}_{\psi=0}^{\theta=0} : \text{length of } \theta = 0 \text{ and } \psi = 0 \text{ curve}, \quad (42)$$

$$\mathcal{C}_{\phi=0}^{\theta=\pi/2} : \text{length of } \theta = \frac{\pi}{2} \text{ and } \phi = 0 \text{ curve}, \quad (43)$$

and we define

$$V_1(a) = \max \{ \mathcal{C}_{\psi=0}^{\phi=0}, \mathcal{C}_{\psi=0}^{\theta=\pi/2}, \mathcal{C}_{\psi=0}^{\theta=0}, \mathcal{C}_{\phi=0}^{\theta=\pi/2} \}. \quad (44)$$

Here, we have taken the isometry of the horizon geometry into account as before. If V_1 gives a measure for black hole formation, the properties (i) and (ii) should be satisfied.

The values of \mathcal{C} 's are depicted as functions of a in some cases of m/l^2 in Fig.3. As can be seen in the Fig.3, the maximal one is $\mathcal{C}_{\psi=0}^{\phi=0}$ in all cases, then $V_1 = \mathcal{C}_{\psi=0}^{\phi=0}$. Though V_1 is a monotonic increasing function of a for all cases of m/l^2 , $V_1(a_{\text{cr}})$ becomes much larger than the critical size, $2G_5 M/l$, in the case $m/l^2 \ll 1$.

TABLE I: Inequalities and properties for black hole formation.

Inequality	$V_2 \lesssim 32G_5 M/3$		$V_1 \lesssim 2G_5 M/l$		$W \lesssim 32G_5 M/3$	
Property	(i)	(ii)	(i)	(ii)	(i)	(ii)
$m/l^2 \gg 1$	No	No	Yes	Yes	Yes	Yes
$m/l^2 \ll 1$	Yes	Yes	Yes	No	Yes	Yes

The results of the test are summarized in Table I. We find from this table that V_2 gives an appropriate measure for the criterion of horizon formation only for $m/l^2 \lesssim 1$, while V_1 does only for $m/l^2 \gtrsim 1$.

D. Hybrid condition

Since V_2 works in the cases $m/l^2 \lesssim 1$, and V_1 works in the cases $m/l^2 \gtrsim 1$, we can expect that a combination of V_1 and V_2 provides a good measure in all range of the mass

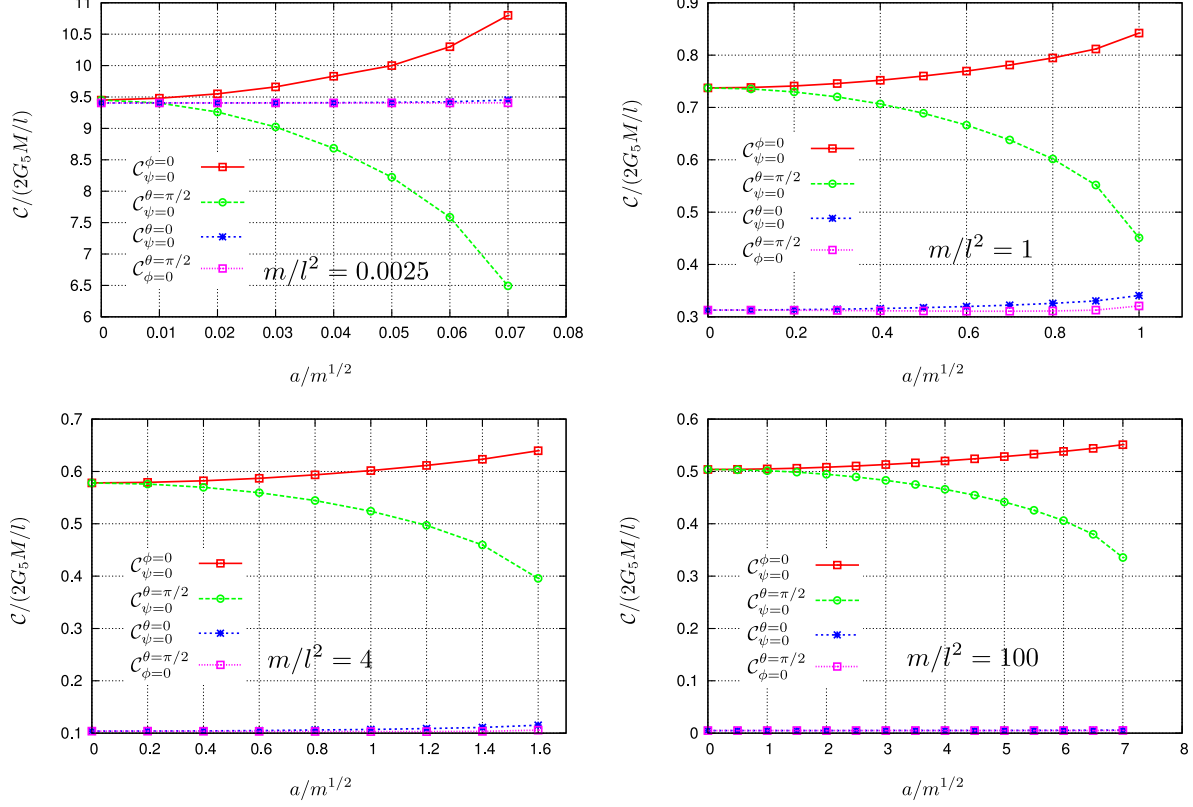


FIG. 3: Length of closed one-dimensional geodesic curves \mathcal{C} on apparent horizons are depicted as functions of $a/m^{1/2}$ in the cases of $m/l^2 = 0.0025, 1, 4$, and 100 . The critical values of $V_1 = \mathcal{C}_{\psi=0}^{\phi=0}$ become much larger than the critical size $2G_5M/l^2$ in the $m/l^2 = 0.0025$ case.

scale for horizon formation in Kaluza-Klein spaces. According to the results in the previous subsection, we can immediately find $V_2 \gg lV_1$ for $m/l^2 \gg 1$ and $V_2 \ll lV_1$ for $m/l^2 \ll 1$. Then we propose a new condition for horizon formation:

$$W \lesssim \frac{32}{3}G_5M \quad (45)$$

with the following definition of W :

$$\frac{1}{W} := \frac{3}{16lV_1} + \frac{1}{V_2}. \quad (46)$$

We plot W as a function of a in Fig.4. We can see from this figure that W satisfies two properties: (i) W is a monotonic increasing function of a ; and (ii) $W(a_{\text{cr}}) \sim \frac{32}{3}G_5M$. Of course, in the asymptotically Euclidean case, i.e., $l \rightarrow \infty$, (45) reduces to the inequality (31), and in the limit $l \rightarrow 0$ it reduces to (39). Then, the condition (45) with (46) is a hybrid of

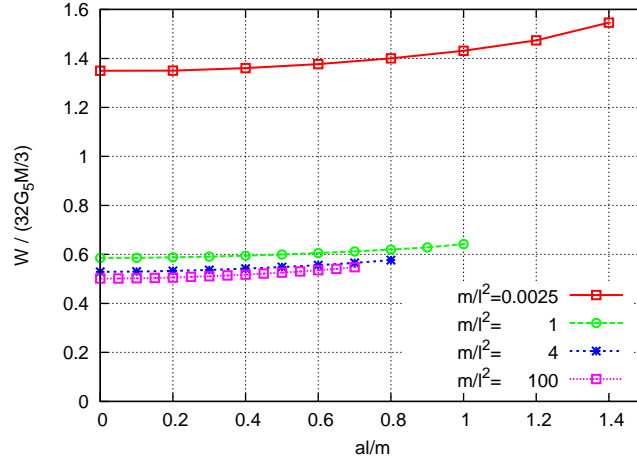


FIG. 4: Values of W are depicted as functions of al/m for the cases of $m/l^2 = 0.0025, 1, 4, 100$. W is a monotonic increasing function of al/m and $W(a_{\text{cr}}) \sim 1$ in all cases.

the four-dimensional hoop conjecture and the five-dimensional hyperhoop conjecture³.

We should note that there is a significant difference between W and V_2 . The hyperhoop V_2 is just a geometrical quantity which represents the typical size of the horizon, while W contains the size of extra dimension l which is related to the asymptotic property.

Extrapolating this idea to general dimensions D , we can consider the following extended version of the hyperhoop conjecture: *Black holes with horizons form in a D -dimensional spacetime when and only when a mass M gets compacted into a region whose n -dimensional volume V_n ($n = 1, 2, \dots, D - 3$) in every direction satisfy*

$$\left(\sum_i^n \frac{1}{\beta_i V_i \prod_k^{D-i-3} l_k} \right)^{-1} \lesssim G_D M, \quad (47)$$

where β_i are numerical factors and l_n , ($l_1 \leq l_2 \cdots \leq l_{D-4}$) are the compactification scales of each compactified direction, and the n -dimensional volume means the volume of a n -dimensional closed submanifold of a spacelike hypersurface.

³ In fig.4, W does not tend to 1 in $a \rightarrow 0$ limit even though $m \ll l^2$ or $m \gg l^2$. This would be because the horizon topology is not S^3 . Furthermore, the geometry of the initial surfaces differs from a time slice of squashed black hole solutions in the case $m \gg l^2$.

IV. CROSS SECTION OF BLACK HOLE PRODUCTION IN KALUZA-KLEIN SPACES

In this section, on the basis of the new conjecture proposed in the previous section, we discuss the mass dependence of the cross section of black hole formation due to collision of particles.

A. Case of five dimensions

In the five-dimensional case, the black hole formation condition could be given by (45) with (46). In the case of the collision of particles, we expect that the shape of a black hole is not highly elongated in our four dimensions. Then, the cross section σ_p likely to be given by

$$\sigma_p \sim \pi \left(\frac{V_1}{2\pi} \right)^2 \sim \pi \left(\frac{V_2}{4\pi} \right). \quad (48)$$

Based on this assumption, we can estimate σ_p using (45) with (46) as follows. We replace V_1 and V_2 in (46) by using (48), and set $W = \frac{32}{3}G_5M$, then we get

$$\frac{3}{16l\sqrt{4\pi\sigma_p}} + \frac{1}{4\sigma_p} = \frac{3}{32G_5M}. \quad (49)$$

We can solve this equation with respect to σ_p as

$$\sigma_p/l^2 = \frac{8}{3}G_5M/l^2 + \frac{1}{2\pi}(G_5M/l^2)^2 + \frac{1}{2\pi}G_5M/l^2 \sqrt{\frac{32\pi}{3}G_5M/l^2 + (G_5M/l^2)^2}. \quad (50)$$

The value of σ_p is plotted in Fig.5 as a function of G_5M/l^2 . The numerical values of $V_1^2(a_{cr})/4\pi$ for five different values of $m/l^2 = G_5M/(6\pi l^2)$ are superposed in Fig.5.

We can see a transition of power-law dependence of the cross section on the mass scale from $\sigma_p \propto G_5M$ to $\sigma_p \propto (G_5M)^2$ as M increases. The total mass can be regarded as the center of mass energy in the high energy particle collision. The mass dependence of the cross section comes from the mass dependence of size and shape of the horizons.⁴ Although the values of $V_1^2(a_{cr})/4\pi$ are somewhat deviated from the line of σ_p , these are still same order, and the plots follow the transition of σ_p .

⁴ Actually, we can see the mass dependence of apparent horizon size of the initial data in the $a = 0$ case as shown in Appendix B.

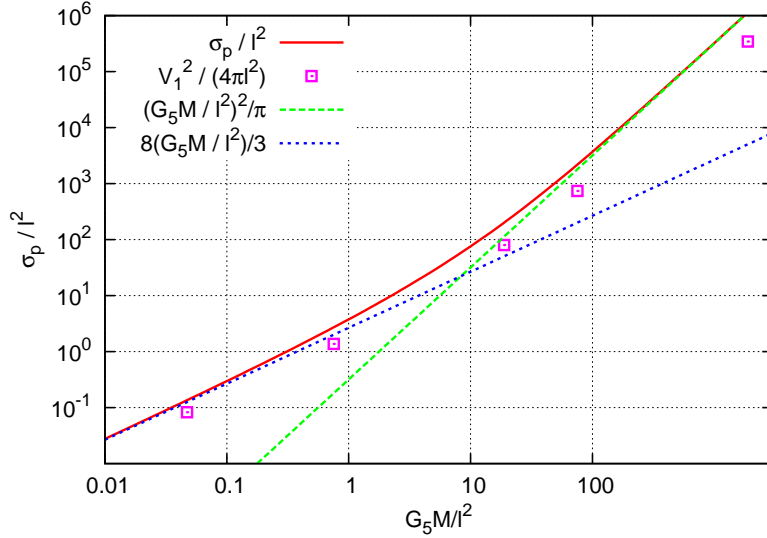


FIG. 5: The cross section σ_p of five-dimensional Kaluza-Klein black hole formation which is estimated by the hybrid hoop conjecture is depicted as a function of $G_5 M / l^2$.

B. Case of arbitrary dimension

It is interesting to consider a compactification which has different compactification scales. At least, one of these scales contributes to resolving the hierarchy problem, then it should be much larger than the length scale $\sim 10^{-17}\text{cm}$ for TeV gravity scenarios. We can consider a possibility that other compactification scales take intermediate scales. It would be possible that the energy scale of colliding particles may be the same order of the intermediate compactification scales. In this case, we can obtain the information of the numbers of compactified dimensions and the size of them from the mass dependence of the black hole production rate.

To see this, let us consider that compact n_* dimensions have a size l_* , compact n_L dimensions, which would contribute to resolving the hierarchy problem, are larger than l_* , and other compact dimensions are much smaller than l_* . For a black hole with mass M , if its horizon size is smaller than l_* , the effective dimension is $D_{\text{eff}} = 4 + n_L + n_*$, i.e., we should consider the black hole is in a $(4 + n_L + n_*)$ -dimensional spacetime, where we assumed the sizes of other extra dimensions are much smaller than the horizon size. On the other hand, if the horizon is larger than l_* , the effective dimension is $D_{\text{eff}} = 4 + n_L$ because n_* dimensions become ineffective, i.e., the black hole is effectively in $(4 + n_L)$ dimensions.

The condition of black hole formation in D_{eff} dimensions would be given by

$$V_{D_{\text{eff}}-3} \lesssim \alpha_{D_{\text{eff}}} G_{D_{\text{eff}}} M, \quad (51)$$

where

$$\alpha_D = \frac{16\pi}{D-2} \frac{\Omega_{D-3}}{\Omega_{D-2}}, \quad (52)$$

and Ω_D is the D -dimensional area of the unit D -sphere. For smaller black holes, $D_{\text{eff}} = 4 + n_L + n_*$, and for larger black holes $D_{\text{eff}} = 4 + n_L$. As is done in (48), we estimate σ_p by

$$\begin{aligned} V_{(1+n_L+n_*)} &= \left(\frac{\sigma_p}{4\pi}\right)^{(1+n_L+n_*)/2} \Omega_{(1+n_L+n_*)}, \\ V_{(1+n_L)} &= \left(\frac{\sigma_p}{4\pi}\right)^{(1+n_L)/2} \Omega_{(1+n_L)}. \end{aligned} \quad (53)$$

Then, the mass dependence of the cross section for small black holes is

$$\sigma_p \simeq 4\pi \left(\frac{16\pi G_{(4+n_L+n_*)}}{(2+n_L+n_*)\Omega_{(2+n_L+n_*)}} M \right)^{2/(1+n_L+n_*)}, \quad (54)$$

and for large black holes

$$\sigma_p \simeq 4\pi \left(\frac{16\pi G_{(4+n_L)}}{(2+n_L)\Omega_{(2+n_L)}} M \right)^{2/(1+n_L)}. \quad (55)$$

Therefore, by $\log \sigma_p - \log M$ plot, we obtain directly the numbers of dimensions $(1+n_L+n_*)$, $(1+n_L)$, and effective gravitational constants $G_{(4+n_L+n_*)}$ and $G_{(4+n_L)}$. Then, we can estimate l_* and the volume of the extra dimensions larger than l_* , say Vol_{n_L} , by

$$\begin{aligned} 2\pi l_* &\sim \left(\frac{G_{(4+n_L+n_*)}}{G_{(4+n_L)}} \right)^{1/n_*}, \\ \text{Vol}_{n_L} &\sim \frac{G_{(4+n_L)}}{G_4}. \end{aligned} \quad (56)$$

From the crossover point M_*, σ_p^* of the cross section from (54) to (55) we can also estimate l_* and Vol_{n_L} as

$$2\pi l_* \sim \left(\frac{2+n_L+n_*}{2+n_L} \frac{\Omega_{(2+n_L+n_*)}}{\Omega_{(2+n_L)}} \right)^{1/n_*} \left(\frac{\sigma_p^*}{\pi} \right)^{1/2}, \quad (57)$$

$$\text{Vol}_{n_L} \sim \frac{(2+n_L)\Omega_{(2+n_L)}}{16\pi G_4 M_*} \left(\frac{\sigma_p^*}{\pi} \right)^{(1+n_L)/2}. \quad (58)$$

As a demonstration, in the case of total dimension $D = 10$ and compact dimension 6, we show the M dependence of σ_p which is defined by using Eqs.(53) and the saturated inequality (47) (see Fig.6).

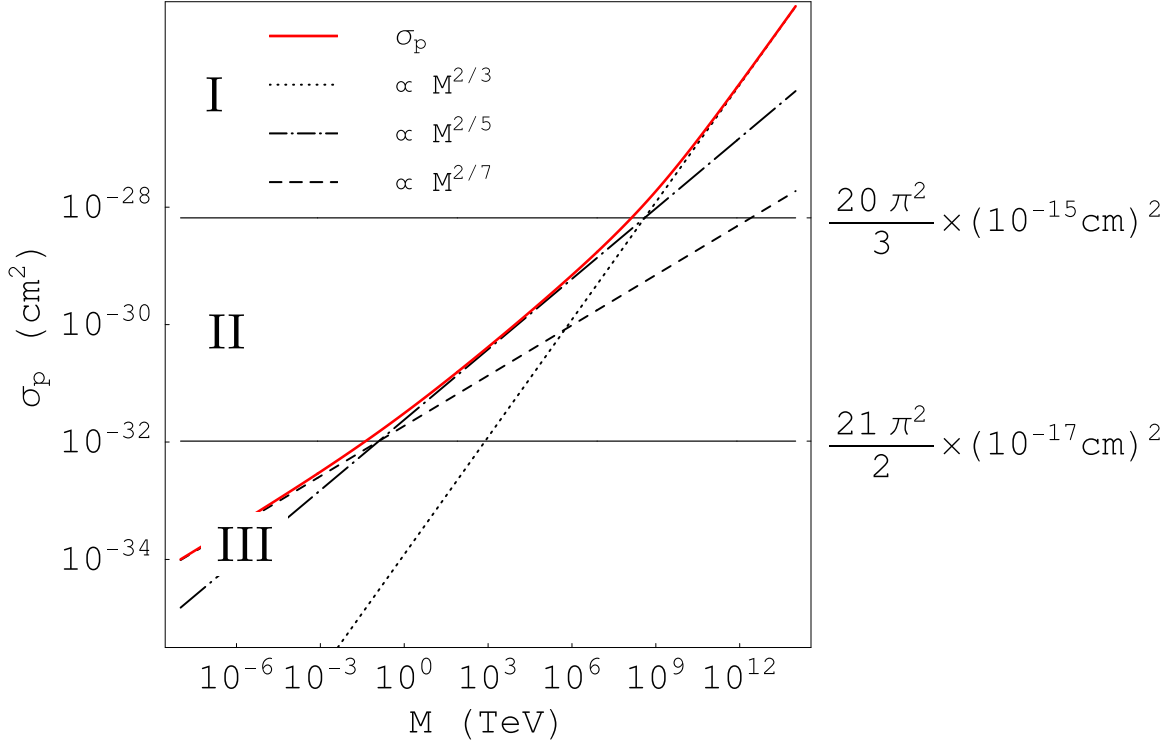


FIG. 6: The cross section of black hole formation σ_p , which is estimated by the hybrid hoop conjecture, is depicted as a function of the center of mass energy M . The total dimension is 10 and six of them are compactified in each scale. Two directions, which contribute to resolving the hierarchy problem, are compactified in 10^{-2}cm . Another two of the compactified directions have the size 10^{-15}cm , and the remaining two dimensions have the size 10^{-17}cm . The power exponent of the cross section changes around $\sigma_p = 20\pi^2/3 \times (10^{-15}\text{cm})^2$ and $\sigma_p = 21\pi^2/3 \times (10^{-17}\text{cm})^2$ which are given by Eq.(57). The numbers of effective dimensions D_{eff} are given by 6, 8, 10 in the regions I, II, and III, respectively. The power exponent of σ_p in each region is given by $2/(D_{\text{eff}} - 3)$ on the mass scale.

V. SUMMARY AND DISCUSSIONS

We have constructed initial data sets of the gravitational field produced by two-point masses which represent Kaluza-Klein spaces in the asymptotically far region. These systems are characterized by the size l of the compactified dimension, the mass scale m , and the separation of the particles a . By using these initial data we have investigated the geometry of apparent horizons and condition of horizon formation in the five-dimensional Kaluza-Klein

spacetimes. Furthermore, we have discussed the cross section of black hole production by two particle systems.

Thorne's original hoop conjecture represents that an one-dimensional closed curve V_1 gives the condition of black hole formation in four-dimensional spacetimes. The hyperhoop conjecture represents that a two-dimensional closed surface V_2 gives the condition in five-dimensional spacetimes. These would be true if the spacetimes have asymptotically Euclidean spatial sections. However, we have shown that the hyperhoop conjecture in five dimensions is not valid in the case of an asymptotically Kaluza-Klein space where the size of an extra dimension l is comparable to the scale of the black hole horizon. Instead, we have proposed an alternative condition [see (45)] for horizon formation using a geometrical value W which is defined as the harmonic average of lV_1 and V_2 . The exact definition is given by Eq.(46). Then the new criterion is a hybrid of the four-dimensional hoop conjecture and the five-dimensional hyperhoop conjecture. The quantity W contains not only the quantities V_1 and V_2 but also the size l of the compactified dimension, which is related to the asymptotic geometry.

Using the value of W , we have investigated the mass dependence of the cross section σ_p of black hole formation [see (50)]. As expected, in five dimensions, σ_p is proportional to the mass when the mass scale is much less than the scale of the compact dimension, $G_5 M/l^2 \ll 1$, while σ_p is in proportion to the mass square when the mass scale is much larger than the scale of the compact dimension, $G_5 M/l^2 \gg 1$. We have shown that the transition of σ_p actually occurs. We can obtain the information of the size of extra dimensions by observing the mass dependence of the black hole production rate. If the total dimension is larger than 5, and extra dimensions would have different compactification scales, we expect from the hybrid hoop conjecture that the mass dependence of the black hole production rate tells us the number of large compact dimensions and the each size of them.

Because the larger energy makes the larger size of a black hole, then it gives us the information of larger extra dimensions. Even though the energy scale in laboratories is too small to verify this effect, we hope that the information of the large extra dimensions is given by active phenomena in astrophysics through the mass dependence of the black hole production rate.

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Appendix A: Test of hyperhoop conjecture in asymptotically Euclidean spaces

Let us consider the case of asymptotically Euclidean spaces. We can write the conformally flat induced metric as

$$h_{ij}dx^i dx^j = F^2 \left[dr^2 + r^2 \left(d\theta^2 + \sin^2 \theta \left(d\phi^2 + \sin^2 \phi d\psi^2 \right) \right) \right], \quad (\text{A1})$$

where the range of the angular coordinates is given by $0 \leq \theta \leq \pi$, $0 \leq \phi \leq \pi$, and $0 \leq \psi \leq 2\pi$.

The vacuum Hamiltonian constraint becomes

$$\Delta_{4\text{dE}} F = 0, \quad (\text{A2})$$

where $\Delta_{4\text{dE}}$ is the Laplace operator on the four-dimensional Euclidean metric. A solution of this equation which has two-point sources is given by

$$F = 1 + \frac{2m_1}{r^2 + a^2 - 2ar \cos \theta} + \frac{2m_2}{r^2 + a^2 + 2ar \cos \theta}, \quad (\text{A3})$$

where m_1, m_2 are mass parameters of each particle and a is the separation parameter. For this initial data, we can calculate the ADM mass as

$$G_5 M_{AD} = 3\pi(m_1 + m_2). \quad (\text{A4})$$

Hereafter, we set $m_1 = m_2 = m$. Using the initial data, we obtain apparent horizons $r = r_h(\theta)$ by the same way in Sec.II. We consider the following typical closed geodetic 2-surfaces on a horizon:

$$\mathcal{A}_{\theta=\pi/2} : \text{area of } \theta = \frac{\pi}{2} \text{ surface}, \quad (\text{A5})$$

$$\mathcal{A}_{\psi=0} : \text{area of } \psi = 0 \text{ surface}, \quad (\text{A6})$$

and the hyperhoop $V_2(a)$,

$$V_2(a) = \max \{ \mathcal{A}_{\theta=\pi/2}, \mathcal{A}_{\psi=0} \}. \quad (\text{A7})$$

The values of \mathcal{A} 's are depicted as functions of a/\sqrt{m} in Fig.7.

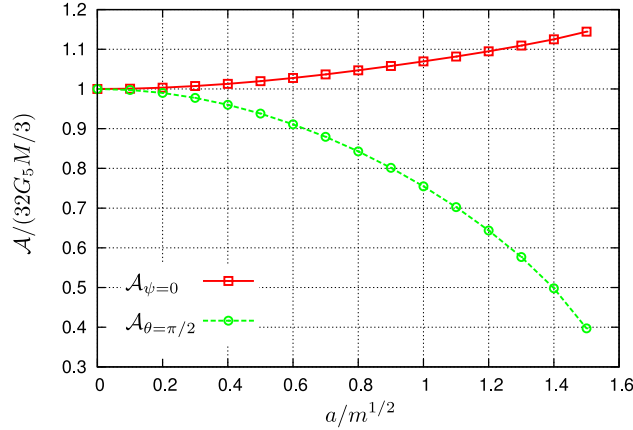


FIG. 7: \mathcal{A} : Areas of the two-dimensional geodesic surfaces on apparent horizons.

As shown in this figure, $V_2 = \mathcal{A}_{\psi=0}$ is a monotonic increasing function of a . In addition, $3V_2(a_{\text{cr}})/(32\pi G_5 M) \sim 1$. That is, $V_2(a)$ satisfies the two properties (i) and (ii) in the text.

Appendix B: Mass dependence of the horizon size in the $a = 0$ case

In the $a = 0$ case, as noted in Sec.II, we can easily find the apparent horizon. We can calculate the areas of the closed geodesic 2-surfaces \mathcal{A} 's and the length of the closed geodesics \mathcal{C} 's on the apparent horizon analytically, as Table II. In Table II, r_h is given by (21) and we have carefully chosen the integral ranges of the angular coordinates for the surfaces and the curves to be closed.

The horizon is a squashed lens space, and the ratio of the size of S^1 fiber to the size of S^2 base is given by

$$\frac{\mathcal{C}_{\psi=0}^{\theta=0}}{\mathcal{C}_{\psi=0}^{\phi=0}} = \left(1 + \frac{l}{r_h}\right)^{-1} \frac{l}{r_h}. \quad (\text{B1})$$

If we consider the case of $m/l^2 \gg 1$, then $r_h \simeq m/l$. The hoop V_1 and the hyperhoop V_2

TABLE II: Closed geodetic surfaces, where $F_{a=0} = 1 + \frac{2m}{lr_h}$, $V_{a=0}^{-1} = 1 + \frac{l}{r_h}$, and $E[x]$ is the complete elliptic integral of second kind defined by $E[x] := \int_0^{\frac{\pi}{2}} \sqrt{1 - x \sin^2 \theta} d\theta$.

Name	$\mathcal{C}_{\psi=0}^{\phi=0}$	$\mathcal{C}_{\psi=0}^{\theta=\pi/2}$	$\mathcal{C}_{\psi=0}^{\theta=0}$	$\mathcal{C}_{\phi=0}^{\theta=\pi/2}$
Definition	$\psi, \phi = 0$	$\theta = \pi/2, \psi = 0$	$\theta = 0, \psi = 0$	$\theta = \pi/2, \phi = 0$
Period	$0 \leq \theta \leq 2\pi$	$0 \leq \phi \leq 2\pi$	$0 \leq \phi \leq 2\pi$	$0 \leq \psi \leq 4\pi$
Length ($a = 0$)	$2\pi r_h F_{a=0} V_{a=0}^{-1/2}$	$2\pi r_h F_{a=0} V_{a=0}^{-1/2}$	$2\pi l F_{a=0} V_{a=0}^{1/2}$	$2\pi l F_{a=0} V_{a=0}^{1/2}$
$m/l^2 \gg 1$	$6\pi m/l$	$6\pi m/l$	$6\pi l$	$6\pi l$
$m/l^2 \ll 1$	$4\pi\sqrt{2m}$	$4\pi\sqrt{2m}$	$4\pi\sqrt{2m}$	$4\pi\sqrt{2m}$

Name	$\mathcal{A}_{\theta=\pi/2}$	$\mathcal{A}_{\phi=0}$	$\mathcal{A}_{\psi=0}$
Definition	$\theta = \pi/2$	$\phi = 0$	$\psi = 0$
Period	$0 \leq \phi \leq 2\pi, 0 \leq \psi \leq 4\pi$	$0 \leq \theta \leq 2\pi, 0 \leq \psi \leq 4\pi$	$0 \leq \theta \leq 2\pi, 0 \leq \phi \leq 2\pi$
Area ($a = 0$)	$4\pi^2 r_h l F_{a=0}^2$	$4\pi^2 r_h l F_{a=0}^2$	$8\pi r_h l F_{a=0}^2 E\left[-\frac{r_h}{l}\left(\frac{r_h}{l} + 2\right)\right]$
$m/l^2 \gg 1$	$36\pi^2 m$	$36\pi^2 m$	$72\pi(m/l)^2$
$m/l^2 \ll 1$	$32\pi^2 m$	$32\pi^2 m$	$32\pi^2 m$

given by (44) and (35) behave as

$$V_1 \simeq \frac{6\pi m}{l}, \quad (\text{B2})$$

$$V_2 \simeq \frac{72\pi^2 m^2}{l^2}. \quad (\text{B3})$$

On the other hand, if we consider the case of $m/l^2 \ll 1$, then $r_h \simeq 2m/l$ and

$$V_1 \simeq \mathcal{C}_{\psi=0}^{\phi=0} \Big|_{a=0} = \mathcal{C}_{\psi=0}^{\theta=\pi/2} \Big|_{a=0} \simeq \mathcal{C}_{\psi=0}^{\theta=0} \Big|_{a=0} = \mathcal{C}_{\phi=0}^{\theta=\pi/2} \Big|_{a=0} \simeq 4\pi\sqrt{2m}, \quad (\text{B4})$$

$$V_2 \simeq \mathcal{A}_{\theta=\pi/2} \Big|_{a=0} = \mathcal{A}_{\phi=0} \Big|_{a=0} \simeq \mathcal{A}_{\psi=0} \Big|_{a=0} \simeq 32\pi^2 m. \quad (\text{B5})$$

They mean that the apparent horizon is round.

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